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Technical Notes

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Heat-Transfer Coefficient for a Long Fin Cooled by Convection and Radiation

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Nomenclature

A_{eff}	=	effective surface area, $\pi D L_{\text{eff}}$, m^2
D	=	diameter, mm
g	=	acceleration of gravity, m/s^2
h	=	combined heat-transfer coefficient, $h_c + h_r$, W/m^2-K
h_c	=	natural convection heat-transfer coefficient, W/m^2-K
h_r	=	radiation heat-transfer coefficient, W/m^2-K
$\bar{h}$	=	average combined heat-transfer coefficient, W/m^2-K
k	=	thermal conductivity of fin, W/m-K
k_f	=	thermal conductivity of fluid, W/m-K
L	=	length, m
L_{eff}	=	effective length, m
m	=	dimensionless fin parameter
$\bar{m}$	=	average dimensionless fin parameter
Nu_D	=	Nusselt number, $h_c D/k_f$
P	=	perimeter, m
Pr	=	Prandtl number, ν/α
$\dot{Q}$	=	rate of heat transfer, W
Ra_D	=	Rayleigh number, $g\beta(T - T_\infty)D^3/\nu\alpha$
T	=	temperature along the fin, °C
T_f	=	mean film temperature, $(T + T_\infty)/2$, °C
U	=	uncertainty
x	=	axial coordinate, m
α	=	thermal diffusivity, m^2/s
ϵ	=	emissivity
θ	=	excess temperature, $T - T_\infty$, °C
ν	=	kinematic viscosity, m^2/s
σ	=	Stefan–Boltzmann constant, W/m^2-K^4
χ	=	dimensionless axial coordinate, x/L

Subscripts

b	=	base
s	=	surface
surr	=	surroundings
∞	=	ambient

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Introduction

THE physical situation considered in this study is a horizontal cylindrical rod or pin fin that is sufficiently long so that heat loss from the tip is negligible. Heat is removed from the surface of the fin via natural convection and radiation. In the case of natural convection and radiation, the heat-transfer coefficient depends on the local surface temperature that varies along the fin. The effect of a temperature-dependent heat-transfer coefficient has been investigated.[1–8] Several studies[1–5] have applied a rather complicated mathematical analysis to the situation in which the heat-transfer coefficient has a power-law temperature dependence and the exponent determines the heat-loss mechanism, for example, free convection, forced convection, boiling, or radiation. As an alternative to the complicated mathematics, several researchers[6–8] have employed numerical solutions. Assis et al.[6] used the finite difference method to study the transient heating and cooling of spines exposed to free convection and radiation. The focus of their study was the temperature distribution, and the numerical results were compared to measurements. Recently, Mokheimer performed a finite difference study first for annular fins[7] and later for a variety of fin types.[8] The convection heat-transfer coefficient was calculated with a correlation.

The average or representative heat-transfer coefficient $\bar{h}$ is useful to describe this situation; however, how this representative value is obtained is not clearly explained. (In this study, a bar over a quantity denotes an average value over the length, and the term "average" is used in a very general sense to indicate a representative value that can be used to characterize the physical situation or in an analytical expression.) The experimental determination of the heat-transfer coefficient has been the topic of a series of papers.[9–11] These studies used a mathematical model for both transient and steady-state temperature measurements to calculate $\bar{h}$. A significant conclusion reached in these studies is that the theory describing this situation is not well understood.

The objective of the present study is to determine the average heat-transfer coefficient for a long horizontal fin rod cooled by natural convection and radiation using both temperature measurements and numerical prediction. The temperature along the fin is calculated using a general finite difference approach that accounts for natural convection with a published correlation and for radiative heat transfer with a simple model.[12] This predicted temperature distribution is then used to calculate the average heat-transfer coefficient by two methods: 1) integration of the combined heat-transfer coefficient over the portion of the fin that loses heat and 2) a heat balance with the average temperature along the fin rod. To confirm the numerical results, temperature measurements are fit to an exponential model using a least-squares approach, and an average heat-transfer coefficient is obtained. Agreement between the experimental results and the heat-balance method prediction is very good.

Governing Equations

Consider a cylindrical pin fin with a base maintained at constant temperature T_b that extends into a fluid of temperature T_∞. With the assumption of one-dimensional heat conduction along the fin and steady-state operation, an energy balance yields

$$\frac{d^2T}{d^2x} - \frac{4h}{kD}(T - T_\infty) = 0 \tag{1}$$

Table 1 Fin data and calculated parameters

							$\bar{h}$, W/m^2-K		
Fin	D, mm	L, m	T_b, °C	T_∞, °C	L_{eff}, m	$\dot{Q}_s$, W	Exper. Eq. (8)	h-integration Eq. (11)	Heat balance Eq. (12)
1	3.18	0.685	92.8	20.8	0.39	0.90	15.0	12.5	15.2
2	6.35	0.685	96.0	21.0	0.57	2.38	12.6	10.2	12.2
3	9.53	0.90	89.8	20.9	0.75	3.77	11.4	8.9	10.7

where h is the combined heat-transfer coefficient that accounts for natural convection and radiation.

The natural convection heat-transfer coefficient is assumed to depend on the local surface temperature. For a horizontal, isothermal cylinder, the Nusselt-number correlation recommended by Churchill and Chu[13] for pure natural convection is

$$Nu_D = \frac{h_c D}{k_f} = 0.36 + \frac{0.518 Ra_D^{\frac{1}{4}}}{[1 + (0.559/Pr)^{9/16}]^{\frac{4}{9}}} \tag{2}$$

which is valid for $10^{-6} < Ra_D < 10^9$. All thermodynamic properties of the fluid are evaluated at the mean film temperature. Note that h_c is a circumferentially averaged value but to avoid confusion with the axially averaged value the term average is not used. The radiation heat-transfer coefficient is given by

$$h_r = \epsilon\sigma\left(T^2 + T_{\text{surr}}^2\right)(T + T_{\text{surr}}) \tag{3}$$

In this study, the surroundings are assumed to be completely absorbing and at the same temperature as the ambient air, while k and ϵ are assumed to be independent of the fin temperature and thus constant.

The temperature at the fin base is specified, so that the boundary condition at $x = 0$ is given. An energy balance at the fin tip requires that the heat transfer conducted through the tip surface is equal to the heat transfer from the tip caused by convection and radiation. However, in this study the fins under consideration are assumed to be sufficiently long so that the heat transfer from the tip is negligible. Equation (1) can be written in a more general form with the introduction of the excess temperature and the dimensionless axial coordinate, that is,

$$\frac{d^2\theta}{d^2\chi} - m^2\theta = 0 \tag{4}$$

with the dimensionless fin parameter $m \equiv L(4h/kD)^{1/2}$. The boundary conditions are expressed in mathematical form as

$$\theta|_{\chi=0} = \theta_b \qquad \text{and} \qquad \left.\frac{d\theta}{d\chi}\right|_{\chi=1} = 0 \tag{5}$$

Equations (4) and (5) constitute a nonlinear, two-point boundary-value problem—the problem is nonlinear because m (actually h) varies with temperature.

The standard approach in the development is to assume a uniform and constant heat-transfer coefficient $\bar{h}$, which leads to an analytical solution.[14] For a long fin ($L \to \infty$) with a uniform heat-transfer coefficient, the analytical temperature distribution is given as

$$\theta(\chi) = \theta_b e^{-\bar{m}\chi} \tag{6}$$

where $\bar{m} \equiv L(4\bar{h}/kD)^{1/2}$ is a representative value of the fin parameter.

Temperature Distribution

Experimental

The temperature along the fins was measured using the setup shown in Fig. 1. The apparatus consisted of a heating plate mounted on a stand to which the rod was attached. The base plate can be heated and maintained at a constant temperature by adjusting the level of electrical energy input to a heating pad using a variable voltage transformer. Air currents in the room were minimized, and thermocouples were used to measure the temperature of the base plate and the ambient air. A detailed description of the apparatus is given in Refs. 12, 15, and 16.

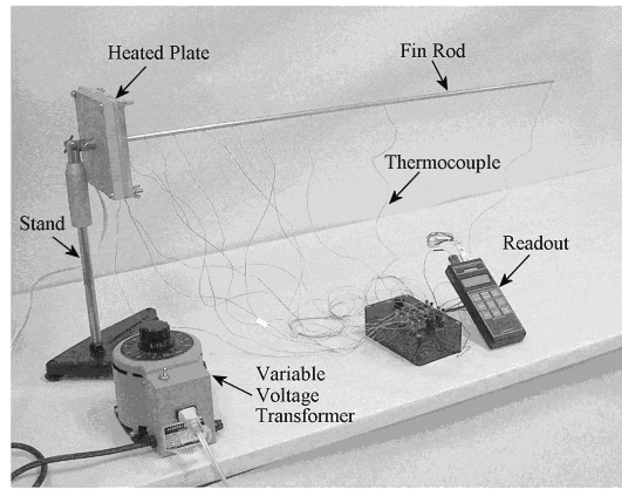

Fig. 1 Schematic of the experimental apparatus.

The fins used in this study were made from Al 2024-T4, which was assumed to have a thermal conductivity independent of temperature and equal to 120 W/m-K.[‡] The emissivity of the aluminum (at surface temperatures between 70 and 100°C) was estimated to be 0.35 using an Omegascope 3000 Infrared Pyrometer. Copper-constanan (Type T) thermocouples were inserted and sealed into small holes in the rod to measure the axial temperature distribution once steady-state conditions were achieved. The base and ambient temperatures for three fins of different diameters are given in Table 1.

Numerical

A standard finite difference scheme (e.g., see Ref. 12) was applied to Eqs. (4) and (5) to determine the temperature along the fin. The model requires the fin geometry (D and L), thermal conductivity, and emissivity to be input, along with the base and ambient temperatures. The properties of air were calculated using a table-look-up interpolation scheme with data from Ref. 14, and Gauss–Seidel iteration was used to solve the equations. A detailed description of the algorithm is given in Ref. 12.

A numerical investigation was performed to study the fins described in Table 1. Following the results of previous computer work,[12] finite difference models with 641 nodal points were used in this study. Agreement between the temperature measurement and the finite difference numerical solution was found to be very good, and inspection of the numerical results indicates that all of the fins can be approximated as long as the heat transfer decays (exponentially) to zero at the tip.[12]

Determination of $\bar{h}$

Experimental

An average value of the heat-transfer coefficient can be found from the temperature measurements following a procedure as described in Refs. 9, 10, and 16. The excess temperature distribution in Eq. (6) can be linearized with division by θ_b and application of the natural logarithm to yield

$$y = \ln(\theta/\theta_b) = -\bar{m}\chi \tag{7}$$

The temperature measurements can then be used to determine $\bar{m}$ using a least-squares approach. Figure 2 shows the linearized

‡Data available online at http://www.matweb.com.

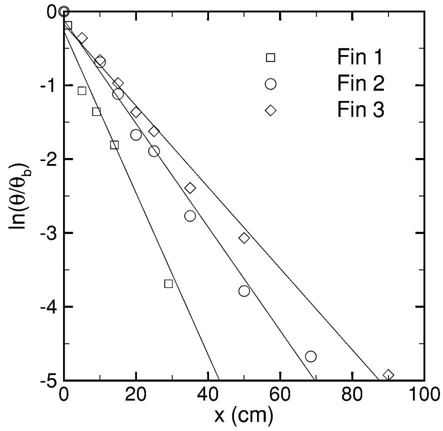

Fig. 2 Linearized temperature distribution along the fin.

temperature measurements for the three fins along with best-fit lines. The linear least-squares approach employed allows for two degrees of freedom; thus, the lines in Fig. 2 do not pass directly through the origin. Note that the approach does not minimize the sum of the squares of the deviations of the nondimensional temperature curve, but rather the deviations of the natural logarithm of the nondimensional temperature curve. This amounts to minimizing the squares of the percentage error between the analytical solution and measurement and thus gives a greater influence to the measurements at the tip of the fin in comparison to those at the base.

Once $\bar{m}$ from the least-squares analysis is determined, the heat-transfer coefficient can be calculated, that is,

$$\bar{h} = \bar{m}^2 k D L^2/4 \tag{8}$$

Values of the average heat-transfer coefficient found with Eq. (8) for each fin are given in Table 1. The calculation of $\bar{h}$ from Eq. (8) is sensitive to the value of $\bar{m}$, which itself depends on the temperature data used. The uncertainty in the calculation of $\bar{h}$ can be estimated, that is,

$$U_{\bar{h}} = \left\{\left[(\bar{m}kL^2D/2)\,U_{\bar{m}}\right]^2 + \left[(\bar{m}^2L^2D/4)\,U_k\right]^2 + \left[(\bar{m}^2kL^2/4)U_D\right]^2 + \left[(\bar{m}^2kLD/2)U_L\right]^2\right\}^{\frac{1}{2}} \tag{9}$$

Values for the quantities in Eq. (9) were assumed (at the 95% confidence level), and the percent uncertainty in $\bar{m}$ was estimated to be 5% based on numerical experiments with several data sets. This resulted in a percent uncertainty in the average heat-transfer coefficient of approximately $\pm 10\%$. The first term in Eq. (9) dominates the uncertainty calculations.

Numerical

The h_c, h_r, and h calculated by the finite difference scheme vary along the fin as shown with the solid lines in Fig. 3. As expected, the heat-transfer coefficient is a maximum at the base where the temperature is the highest and a minimum at the tip where the temperature approaches the ambient. Values of h_c, h_r, and h as predicted by Eqs. (2) and (3) evaluated at the measured temperatures are indicated with symbols. Agreement between the experimental values and numerical predictions are very good. The representative values found from measurements with Eq. (8) are shown as dashed lines. For all three fins, the average value of the heat-transfer coefficient falls between the maximum and minimum value. Of interest from a design standpoint is how to determine the representative or average value of the heat-transfer coefficient along the fin from the numerical solution that can be used to predict the temperature distribution and the rate of heat dissipation.

From a mathematical standpoint, the average value of the heat-transfer coefficient is defined as

$$\bar{h} = \frac{1}{L}\int_0^L h\,\mathrm{d}x \tag{10}$$

For long fins, this integral average approach should be used with caution because a portion of the integration in Eq. (10) might take place over a portion of the fin where no heat transfer occurs. In fact, if the fin is very long the value predicted by Eq. (10) would approach the limiting values at the tip of the fin. Instead of performing the integration over the entire length of the fin, the integral average value of the heat-transfer coefficient is defined as

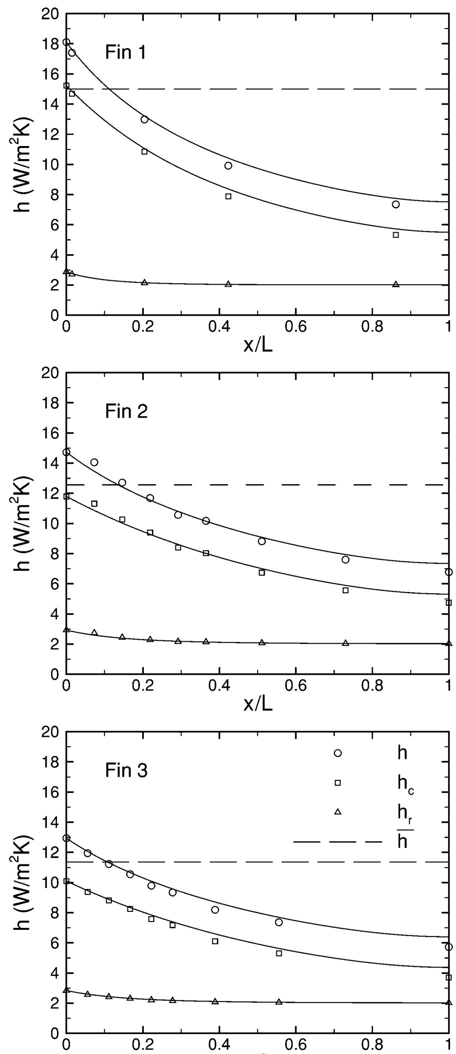

Fig. 3 Variation of the heat-transfer coefficients along the fins.

$$\bar{h} = \frac{1}{L_{\mathrm{eff}}}\int_0^{L_{\mathrm{eff}}} h\,\mathrm{d}x \tag{11}$$

where L_{eff} is defined as the length of the fin from which 99% of the heat is lost. L_{eff} depends on the fin properties D, k, and ϵ, and the temperatures of the base, surroundings, and ambient. The effective lengths and the average heat-transfer coefficients as predicted by Eq. (11) for the three fins are given in Table 1. This integration method simply averages the heat-transfer coefficient over the portion of the fin from which heat is lost. This method does not take into account the fact that most of the heat is lost near the base (see Ref. 12), where h is higher. As a result, this integration method underpredicts the value of $\bar{h}$ as compared with experimental results because most of the heat loss occurs near the base.

An alternate method to find the average heat-transfer coefficient from the temperature distribution makes use of the rate of heat loss from the fin surface $\dot{Q}_s$ and L_{eff}, namely,

$$0.99 \times \dot{Q}_s = \int_0^{L_{\mathrm{eff}}} hP(T - T_\infty)\,\mathrm{d}x = \bar{h}P\int_0^{L_{\mathrm{eff}}}(T - T_\infty)\,\mathrm{d}x = \bar{h}A_{\mathrm{eff}}(\bar{T} - T_\infty) = \bar{h}A_{\mathrm{eff}}\,\bar{\theta} \tag{12}$$

where the average value of the excess temperature is given by

$$\bar{\theta} = \frac{1}{L_{\text{eff}}} \int_0^{L_{\text{eff}}} (T - T_\infty)\,\mathrm{d}x = \frac{1}{L_{\text{eff}}} \int_0^{L_{\text{eff}}} \theta\,\mathrm{d}x \qquad (13)$$

$\bar{\theta}$ is found numerically by trapezoidal integration. The average values of the heat-transfer coefficients found using this heat-balance method are also presented in Table 1. Results from this method are in very good agreement with measured values.

Conclusions

Two approaches to predict the average or representative heat-transfer coefficient for a long, horizontal fin cooled by natural convection and radiation have been presented—the integration method and the heat-balance method. Both approaches make use of a numerically determined temperature distribution that relies on a natural convection correlation and simple radiation model to account for the heat loss. The integration method underpredicts the value of $\bar{h}$ as compared with results from measurements because most of the heat loss occurs near the base where the value of h is higher. The heat-balance method is in very good agreement with the measured values.

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